Práctica trigonometría

1) Compruebe las siguientes identidades:

a) $\csc(x) \cdot \cos\left(x + \frac{\pi}{2}\right) = -1$

$\frac{1}{sen} \cdot \cos\left(x + \frac{\pi}{2}\right)$

$= \frac{1}{senx} \cdot \left(\cos x \cos\frac{\pi}{2} - senx\, sen\frac{\pi}{2}\right)$

$= \frac{1}{senx} \cdot (\cos x \cdot 0 - seny \cdot 1)$

$= \frac{1}{senx} \cdot -senx$

$= \frac{-senx}{senx}$

$= -1$

b) $[\sec(x) + \tan(x)]^2 = \frac{1+sen(x)}{1-sen(x)}$

$\left(\frac{1}{\cos x} + \frac{senx}{\cos}\right)^2$

$= \left(\frac{1+senx}{\cos x}\right)^2$

$= \frac{(1+senx)^2}{\cos^2}$

$= \frac{(1+senx)^2}{1-sen^2x}$

$= \frac{(1+senx)^{\cancel{2}}}{\cancel{(1+senx)}(1-senx)} = \frac{1+senx}{1-senx}$

2) Resuelva las siguientes ecuaciones en el intervalo que se le indica en cada caso:

a) $4\cos^2(x) = \cos(x)$ en $[0, 2\pi]$.

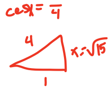
$4\cos^2 x - \cos x = 0$

$\cos(4\cos - 1) = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x = \frac{1}{4}$



$16 - 1^2 = x^2$

$15 = x^2$

$\sqrt{15} = x$

$\arccos\frac{1}{4} = x$

I cuadrante

IV: $2\pi - \arccos\frac{1}{4} = x$

$S = \left\{\frac{\pi}{2}, \frac{3\pi}{2}, \arccos\frac{1}{4}, 2\pi - \arccos\frac{1}{4}\right\}$

b) $2\cos(x) + \sqrt{3} - 2 - \sqrt{3}\sec(x) = 0$ en $\mathbb{R}$.

$2\cos x + \sqrt{3} - 2 - \frac{\sqrt{3}}{\cos} = 0$

$\frac{2\cos^2 x + \sqrt{3}\cos - 2\cos - \sqrt{3}}{\cos x} = 0$

$2\cos^2 x + \sqrt{3}\cos - 2\cos - \sqrt{3} = 0$

$(2\cos^2 x + \sqrt{3}\cos) + (-2\cos - \sqrt{3}) = 0$

$\cos(2\cos + \sqrt{3}) - (2\cos + \sqrt{3}) = 0$

$(\cos - 1)(2\cos + \sqrt{3}) = 0$

$\cos x = 1$ $\quad$ $\cos x = \frac{-\sqrt{3}}{2}$

$x = 0$

II: $\pi - \frac{\pi}{6} = \alpha$
$= \frac{5\pi}{6} = \alpha$

III: $\pi + \frac{\pi}{6} = \alpha$
$= \frac{7\pi}{6} = \alpha$

$S = \{0 + 2k\pi, \frac{5\pi}{6} + 2k\pi, \frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}\}$

c) $\cos(2x) + 2\cos^2\left(\frac{x}{2}\right) = 1$ en $]-2\pi, 0]$

$2\cos^2 x - 1 + 2\left(\sqrt{\frac{1+\cos x}{2}}\right)^2 = 1$

$2\cos^2 x - 1 + \cancel{2}\cdot\frac{1+\cos x}{\cancel{2}} = 1$

$2\cos^2 x - 1 + 1 + \cos x = 1$

$2\cos^2 x + \cos x - 1 = 0$

$2\cos \quad -1$

$\cos \quad 1$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -1$

$x = \pi$

I: $\frac{\pi}{3}$

IV: $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\pi - 2\pi = -\pi$

$\frac{\pi}{3} - 2\pi = -\frac{5\pi}{3}$

$\frac{5\pi}{3} - 2\pi = -\frac{\pi}{3}$

$S = \{-\frac{5\pi}{3}, -\pi, -\frac{\pi}{3}\}$

d) $2sen(2x - \pi) - 1 = 0$ en $[0, 2\pi]$.

$2x - \pi = u$

$2\,sen\,u - 1 = 0$

$sen\,u = \frac{1}{2}$

I: $\frac{\pi}{6}$

II: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$u = \frac{\pi}{6}$ $\quad$ $u = \frac{5\pi}{6}$

$2x - \pi = \frac{\pi}{6}$ $\quad$ $2x - \pi = \frac{5\pi}{6}$

$2x = \frac{\pi}{6} + \pi$ $\quad$ $2x = \frac{5\pi}{6} + \pi$

$2x = \frac{7\pi}{6}$ $\quad$ $2x = \frac{11\pi}{6}$

$x = \frac{7\pi}{12}$ $\quad$ $x = \frac{11\pi}{12}$

$\frac{7\pi}{12} + \pi = \frac{19\pi}{12}$

$\frac{11\pi}{12} + \pi = \frac{23\pi}{12}$

$S = \{\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}\}$

Respuestas

3) Compruebe las siguientes identidades:

c) $\csc(x) \cdot \cos\left(x + \frac{\pi}{2}\right) = -1$

d) $[\sec(x) + \tan(x)]^2 = \frac{1+sen(x)}{1-sen(x)}$

4) Resuelva las siguientes ecuaciones en el intervalo que se le indica en cada caso:

e) $4\cos^2(x) = \cos(x)$ en $[0, 2\pi]$.

f) $2\cos(x) + \sqrt{3} - 2 - \sqrt{3}\sec(x) = 0$ en $\mathbb{R}$.

g) $\cos(2x) + 2\cos^2\left(\frac{x}{2}\right) = 1$ en $]-2\pi, 0]$

h) $2sen(2x - \pi) - 1 = 0$ en $[0, 2\pi]$.