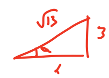
Ejemplo: Halle el valor de:

$$\cos 2(\arctan 1)$$

$\arctan 1$ | $\cos\left(2\cdot\frac{\pi}{4}\right)$
$\tan\alpha = 1$ | $=\cos\left(\frac{\pi}{2}\right)$
$\frac{\pi}{4}$ | $=0$
$\mathrm{I}: \frac{\pi}{4}=\alpha$

$$sen\,2\left(arcsen\frac{3}{\sqrt{13}}\right)$$

$arcsen\frac{3}{\sqrt{13}}=\alpha$ | $sen\,2\alpha$
$sen\,\alpha=\frac{3}{\sqrt{13}}$ | $=2\,sen\alpha\cdot\cos\alpha$
| $=2\cdot\frac{3}{\sqrt{13}}\cdot\frac{2}{\sqrt{13}}$
| $=\frac{12}{13}$

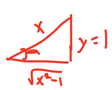


$13-9=x^2$
$4=x^2$
$2=x$
I cuadrante

$$\tan\left(arc\cos\frac{-\sqrt{x^2-1}}{x}\right) = \tan\alpha = \frac{-1}{\sqrt{x^2-1}}$$

$arc\cos\frac{-\sqrt{x^2-1}}{x}=\alpha$

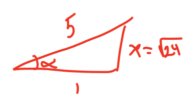
$\cos\alpha=\frac{-\sqrt{x^2-1}}{x}$



$x^2=(\sqrt{x^2-1})^2+y^2$
$x^2=x^2-1+y^2$
$1=y$
II cuadrante

$$\tan 2\left(\arccos\frac{-1}{5}\right)$$

$\cos\alpha=\frac{-1}{5}$ | $\tan 2\alpha$
| $\frac{2\tan\alpha}{1-\tan^2\alpha}$



| $=\frac{2\cdot -\sqrt{24}}{1-(-\sqrt{24})^2}$

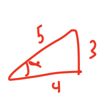
$25-1=x^2$
$\sqrt{24}=x$
II cuadrante

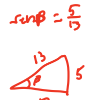
| $=\frac{-2\sqrt{24}}{1-24}$
$=\frac{-2\sqrt{24}}{-23}$
$=\frac{2\sqrt{24}}{23}$

Ejemplo: Calcule $\cos\left(\tan^{-1}\left(\frac{3}{4}\right)+sen^{-1}\left(\frac{5}{13}\right)\right)$

$\arctan\frac{3}{4}=\alpha$ | $arcsen\frac{5}{13}=\beta$ | $\cos(\alpha+\beta)$
$\tan\alpha=\frac{3}{4}$ | $sen\beta=\frac{5}{13}$ | $\cos\alpha\cos\beta-sen\alpha\,sen\beta$
| | $\frac{4}{5}\cdot\frac{12}{13}-\frac{3}{5}\cdot\frac{5}{13}$

I cuadrante





I cuadrante | $=\frac{48}{65}-\frac{15}{65}=\frac{33}{65}$

EJERCICIOS

Calcule $sen^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ $arsen\,\frac{1}{2} = \alpha$ $sen\,\alpha = \frac{1}{2}$ $4 - 1 = x$ $\sqrt{3} = x$ $\alpha = \frac{\pi}{6}$ $sen\,x = \frac{1}{2}$ $I: \frac{\pi}{6}$	Calcule $\cos^{-1}(-1) = \pi$ $\cos\alpha = -1$ $\alpha = \pi$
Calcule $sen^{-1}\left(sen\left(-\frac{\pi}{3}\right)\right) = -\frac{\pi}{3}$ $sen\,\frac{-\sqrt{3}}{2} = x$ $I: 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ $\frac{5\pi}{3} - 2\pi = x$ $-\frac{\pi}{3} = x$ $= \frac{-\pi}{3}$ $-sen\,\frac{\pi}{3}$ | $arsen\,\frac{-\sqrt{3}}{2}$ $sen\,\alpha = \frac{\sqrt{3}}{2}$ $-\frac{\sqrt{3}}{2} = sen\,\frac{-\pi}{3}$ $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$ $\frac{5\pi}{3} - 2\pi = \frac{-\pi}{3}$	Calcule $sen^{-1}\left(sen\left(\frac{2\pi}{3}\right)\right) = \frac{\pi}{3}$ $\pi - \frac{2\pi}{3} = \frac{\pi}{3}$ $arcsen\left(\frac{\sqrt{3}}{2}\right)$ $sen\,x = \frac{\sqrt{3}}{2}$ $I: \frac{\pi}{3}$ $= \frac{\pi}{3}$ $sen\,\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ | $arcsen\left(\frac{\sqrt{3}}{2}\right)$ $\pi - \frac{2\pi}{3} = \frac{\pi}{3}$ | $sen\,x = \frac{\sqrt{3}}{2}$ $\frac{\pi}{3} = sen\,x$

Calcule $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ $\cos x = \frac{-\sqrt{3}}{2}$ $II: \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\cos x = \frac{-\sqrt{3}}{2}$ $\pi - \alpha = \frac{\pi}{6}$ $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$	Calcule $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) = \frac{-\pi}{6}$ $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$ $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{2}{2\sqrt{3}} = \frac{-1}{\sqrt{3}}$ $\tan x = \frac{-1}{\sqrt{3}}$ $x = \frac{\pi}{6}$ $IV: 2\pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\frac{5\pi}{6} - 2\pi = \frac{-\pi}{6}$ $= -\frac{\pi}{6}$ $\frac{-\pi}{6} = \arctan\left(\tan\frac{5\pi}{6}\right)$
Determine $\tan(\arccos(x)) = \frac{\sqrt{1-x^2}}{x}$ IV $\arccos(x) = \alpha$ $\cos(\alpha) = x$ $y = \sqrt{1-x^2}$ $x^2 + y^2 = 1$ $y = \sqrt{1-x^2}$	Calcule $sen\left(2\cos^{-1}\left(\frac{1}{2}\right)\right) = \frac{\sqrt{3}}{2}$ I $2(\arccos(\frac{1}{2}))$ $\cos\alpha = \frac{1}{2}$ $\frac{\pi}{3} = \alpha$ $sen\left(\frac{2\pi}{3}\right)$ $\pi - \frac{2\pi}{3} = \frac{\pi}{3}$

Ejemplo adicional:

Resuelva la siguiente eccuación $tan^2x + 2tanx - 8 = 0$ $[0, 2\pi]$

Restricciones

$\cos x = 0$

$\frac{2k+1}{2}\pi$

$\tan$ $\quad 4$

$\tan$ $\quad -2$

$(\tan + 4)(\tan - 2) = 0$

$S = \{\arctan 2, \arctan(-4) + \pi, \arctan(-4), \arctan(2) + \pi\}$

$\tan x = -4$ | $\tan = 2$

$x = \arctan(-4)$ | $x = \arctan(2)$

IV cuadrante | I cuadrante

$]-\frac{\pi}{2}, 0[$ | $]0, \frac{\pi}{2}[$

$IV: x = \arctan(-4) + 2\pi$ | $I: \arctan 2 = x$

$II: x = \arctan(-4) + \pi$ | $III: \arctan 2 + \pi = x$

Resuelva la siguiente eccuación $2senx\ tanx - tanx - 10\ senx + 5 = 0\ \ en\ \mathbb{R}$

Restricciones

$\cos x \neq 0$

$\frac{2k+1}{2}\pi$

$(2\text{senx}\tan x - \tan x) + (-10\,\text{sen}\,x + 5) = 0$

$\tan x(2\text{sen} - 1) - 5(2\text{sen} - 1) = 0$

$(\tan x - 5)(2\text{sen} - 1) = 0$

$\tan x = 5$

$\arctan 5 = x$

I cuadrante

III: $\arctan 5 + \pi = x$

$\text{sen}\,\frac{1}{2} = x$

I: $x = \frac{\pi}{6}$

II: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\frac{1}{2}$

$S = \{\arctan(5) + 2k\pi,\ \arctan(5) + \pi + 2k\pi,\ \frac{\pi}{6} + 2k\pi,\ \frac{5\pi}{6} + 2k\pi,\ k \in \mathbb{Z}\}$