Resuelva las siguientes inecuaciones:

1) $x - 5(x+2) \geqslant -2(2x+6)$ R/ $S = \mathbb{R}$.

2) $\dfrac{-2}{y-3} \geqslant \dfrac{1}{y} + \dfrac{6}{9-y^2}$ R/ $S = \left]-\infty, -3\right[\cup \left[-\sqrt{3}, 0\right[\cup \left[\sqrt{3}, 3\right[$.

3) $\dfrac{2x}{x^2-1} + \dfrac{5}{x+1} \geqslant \dfrac{x}{x-1}$ R/ $S =]-1, 5] \setminus \{1\}$.

4) $\dfrac{-3}{x-2} - \dfrac{1}{(x-1)^2} \leqslant \dfrac{1}{x-2}$ $S = \left[\dfrac{7-\sqrt{17}}{8}, 1\right[\cup \left]1, \dfrac{7+\sqrt{17}}{8}\right[\cup]2, +\infty[$.

5) $3 - 7|1-5x| > -25$ R/ $S = \left]-\frac{3}{5}, 1\right[$.

6) $\dfrac{(-4x^3 - 3x^2 + 7x)(x^2+1)}{16x^2 - 49} \geqslant 0$ R/ $S = \left]-\infty, \frac{-7}{4}\right[\cup \left]\frac{-7}{4}, 0\right] \cup \left[1, \frac{7}{4}\right[$.

7) $7 - 2|2-4w| - 3 < 1$ R/ $S = \left]-\infty, \frac{1}{8}\right[\cup \left]\frac{7}{8}, \infty\right[$.

8) $\dfrac{3-x}{x-2} < \dfrac{x-5}{1-x}$ R/ $S =]1, 2[\cup \left]\dfrac{7}{3}, +\infty\right[$.

9) $-x^3 + 2x^2 + 5x - 6 \geqslant 0$ R/ $S =]-\infty, -2] \cup [1, 3]$.

1) $x - 5(x+2) \geqslant -2(2x+6)$

$$x - 5x - 10 \geq -4x - 12$$

$$-4x - 10 \geq -4x - 12$$

$$-10 \geq -12$$

$$S = \mathbb{R}$$

2) $\dfrac{-2}{y-3} \geqslant \dfrac{1}{y} + \dfrac{6}{9-y^2}$

$$\frac{-2}{y-3} \geq \frac{1}{y} + \frac{6}{(3+y)(3-y)}$$

$$\frac{2}{3-y} \geq \frac{1}{y} + \frac{6}{(3+y)(3-y)}$$

$$\frac{2 \cdot y(3+y)}{y(3+y)(3-y)} \geq \frac{1(3+y)(3-y) + 6 \cdot y}{y(3+y)(3-y)}$$

$$\frac{6y + 2y^2}{y(3+y)(3-y)} \geq \frac{9 - y^2 + 6y}{y(3+y)(3-y)}$$

$$\frac{6y + 2y^2 - 9 + y^2 - 6y}{y(3+y)(3-y)} \geq 0$$

$$\frac{3y^2 - 9}{y(3+y)(3-y)} \geq 0$$

$$\frac{3(y^2 - 3)}{y(3+y)(3-y)} \geq 0$$

$$\frac{3(y-\sqrt{3})(y+\sqrt{3})}{y(3+y)(3-y)} \geq 0$$

0 -3 3

$-\infty$ -3 $-\sqrt{3}$ 0 $\sqrt{3}$ 3 $+\infty$

	$]-\infty,-3[$	$]-3,-\sqrt{3}[$	$]-\sqrt{3},0[$	$]0,\sqrt{3}[$	$]\sqrt{3},3[$	$]3,+\infty[$
$y-\sqrt{3}$	−	−	−	− 0	+	+
$y+\sqrt{3}$	−	− 0	+	+	+	+
y	−	−	− 0	+	+	+
$3+y$	− 0	+	+	+	+	+
$3-y$	+	+	+	+	+ 0	−
	+	−	+	−	+	−

$$S =]-\infty, 3[\cup [-\sqrt{3}, 0[\cup [\sqrt{3}, 3[$$

3) $\dfrac{2x}{x^2-1} + \dfrac{5}{x+1} \geqslant \dfrac{x}{x-1}$

$$\frac{2x}{(x+1)(x-1)} + \frac{5}{x+1} \geq \frac{x}{x-1}$$

$$\frac{2x + 5(x-1) - x(x+1)}{(x+1)(x-1)} \geqslant 0$$

$$\frac{2x + 5x - 5 - x^2 - x}{(x+1)(x-1)} \geqslant 0$$

$$\frac{-x^2 + 6x - 5}{(x+1)(x-1)} \geqslant 0 \qquad \begin{matrix} -x^2 + 6x - 5 \\ x \quad\quad -5 \\ -x \quad\quad 1 \end{matrix}$$

$$\frac{(x-5)(1-x)}{(x+1)(x-1)} \geq 0$$

$-\infty$ -1 1 5 $+\infty$

	$]-\infty,-1[$	$]-1,1[$	$]1,5[$	$]5,+\infty[$
$x-5$	−	−	− 0	+
$1-x$	+	+ 0	−	−
$x+1$	− 0	+	+	+
$x-1$	−	− 0	+	+
	−	+	+	−

$$S =]-1, 1[\cup]1, 5]$$

$$S =]-1, 5] - \{1\}$$

4) $\dfrac{-3}{x-2} - \dfrac{1}{(x-1)^2} \leqslant \dfrac{1}{x-2}$

$$\frac{-3}{(x-2)} - \frac{1}{(x-1)^2} - \frac{1}{x-2} \leq 0$$

$$\frac{-3(x-1)^2 - 1(x-2) - 1(x-1)^2}{(x-1)^2(x-2)} \leq 0$$

$$\frac{-3(x^2 - 2x + 1) - x + 2 - 1(x^2 - 2x + 1)}{(x-1)^2(x-2)} \leq 0$$

$$\frac{-3x^2 + 6x - 3 - x + 2 - x^2 + 2x - 1}{(x-1)^2(x-2)} \leq 0$$

$$\frac{-4x^2 + 7x - 2}{(x-1)^2(x-2)} \leq 0$$

$$\frac{-(4x^2 - 7x + 2)}{(x-1)^2(x-2)} \leq 0$$

$$\frac{4x^2 - 7x + 2}{(x-1)^2(x-2)} \geq 0$$

$$\Delta = 49 - 4 \cdot -4 \cdot -2$$

$$\Delta = 49 - 32$$

$$\Delta = 17$$

$$\frac{-7 \pm \sqrt{17}}{-8} = x$$

$$x_1 = \frac{7 - \sqrt{17}}{8} \qquad x_2 = \frac{7 + \sqrt{17}}{8}$$

$$\frac{(x - x_1)(x - x_2)}{(x-1)^2(x-2)} \geqslant 0$$

$-\infty$ x_1 $\frac{7-\sqrt{17}}{8}$ 1 x_2 $\frac{7+\sqrt{17}}{8}$ 2 $+\infty$

	$]-\infty,x_1[$	$]x_1,1[$	$]1,x_2[$	$]x_2,2[$	$]2,+\infty[$
$x-x_1$	− 0	+	+	+	+
$x-x_2$	−	−	− 0	+	+
$(x-1)^2$	+	+ 0	+	+	+
$x-2$	−	−	−	− 0	+
	−	+	+	−	+

$$S =]-\infty, -2[\cup \left[\frac{7-\sqrt{17}}{8}, \frac{7+\sqrt{17}}{8}\right] - \{1\}$$

$$S = \left[\frac{7-\sqrt{17}}{8}, 1\right[\cup \left]1, \frac{7+\sqrt{17}}{8}\right] \cup]2, +\infty[$$

5) $3 - 7|1 - 5x| > -25$

$-7|1-5x| > -28$

$|1-5x| < 4$

$-4 < 1-5x < 4$

$-5 < -5x < 3$

$1 > x > \frac{-3}{5}$

$S = \left]\frac{-3}{5}, 1\right[$

6) $\dfrac{(-4x^3 - 3x^2 + 7x)(x^2+1)}{16x^2 - 49} \geqslant 0$

$4x \quad 7$

$x \quad -1$

$$\frac{-x(4x^2-3x+7)(x^2+1)}{(4x+7)(4x-7)} \geqslant 0$$

$$\frac{-x(4x+7)(x-1)(x^2+1)}{(4x+7)(4x-7)} \geqslant 0$$

	$-\infty$	$\frac{-7}{4}$		0		1		$\frac{7}{4}$	$+\infty$
$-x$	+		+	0	−		−		−
$4x+7$	−	0	+		+		+		+
$x-1$	−		−		−	0	+		+
x^2+1	+		+		+		+		+
$4x+7$	−	0	+		+		+		+
$4x-7$	−		−		−		−	0	+
	+		+		−		+		−

$S = \left]-\infty, \frac{-7}{4}\right[\cup \left]\frac{-7}{4}, 0\right] \cup \left[1, \frac{7}{4}\right]$

$7 - 2|2 - 4w| - 3 < 1$

$-2|2-4w| < 1+3-7$

$-2|2-4w| < -3$

$|2-4w| > \frac{3}{2}$

$2-4w > \frac{3}{2}$	$2-4w < \frac{-3}{2}$
$2-\frac{3}{2} > 4w$	$2+\frac{3}{2} < 4w$
$\frac{4}{2}-\frac{3}{2} > 4w$	$\frac{7}{2} < 4w$
$\frac{1}{2} > 4w$	$\frac{7}{2}\cdot\frac{1}{4} < w$
$\frac{1}{2} \div \frac{4}{1} > w$	$\frac{7}{8} < w$
$\frac{1}{8} > w$	$S_{p_2} \left]\frac{7}{8}, +\infty\right[$
$S_{p_1} = \left]-\infty, \frac{1}{8}\right[$	

$S = \left]-\infty, \frac{1}{8}\right[\cup \left]\frac{7}{8}, +\infty\right[$

8) $\dfrac{3-x}{x-2} < \dfrac{x-5}{1-x}$

$$\frac{3-x}{x-2} - \frac{x-5}{1-x} < 0$$

$$\frac{(3-x)(1-x)-(x-5)(x-2)}{(x-2)(1-x)} < 0$$

$$\frac{3-3x-x+x^2-(x^2-2x-5x+10)}{(x-2)(1-x)} < 0$$

$$\frac{3-4x+x^2-x^2+7x-10}{(x-2)(1-x)} < 0$$

$$\frac{3x-7}{(x-2)(1-x)} < 0$$

	$-\infty$	1		2		$\frac{7}{3}$	$+\infty$
$3x-7$	−		−		−	0	+
$x-2$	−		−	0	+		+
$1-x$	+	0	−		−		−
	+		−		+		−

$S = \left]1, 2\right[\cup \left]\frac{7}{3}, +\infty\right[$

7) $-x^3 + 2x^2 + 5x - 6 \geqslant 0$

-1	2	5	-6	1
	-1	1	6	
-1	1	6	0	

$(x-1)(-x^2+x+6) \geq 0$

$-x \quad 3$

$x \quad 2$

$(x-1)(3-x)(x+2) \geq 0$

$x=1 \qquad 3=x \qquad x=-2$

x	$-\infty$	-2		1		3	$+\infty$
$x-1$	$-$		$-$	0	$+$		$+$
$3-x$	$+$		$+$		$+$	0	$-$
$x-2$	$-$	0	$+$		$+$		$+$
	$+$		$-$		$+$		$-$

$S =]-\infty, -2] \cup [1, 3]$